THERMAL CONDUCTIVITY OF MOIST POROUS SOLIDS

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A structural model and the method of computing the effective thermal conductivity of porous moist solid materials are proposed.

A simplified model of the structure (Fig. 1) of a moist material and an approximate method of computation were proposed in [1] for the prognosis of the thermal conductivity of porous solid construction materials.

Although the model [1] is a crude schematization of the structure of a real moist porous solid material, it already took into consideration a significant characteristic of such an object, viz., the presence of dry and moist segments, solid particles and pores, which may have both parallel and series arrangement in relation to the heat flux.

Actually, in every case the heat flux passes through the solid particles (index S) and through the pores filled by liquid W and dry air L, and is also transported by air — vapor mixture LD in the pores. All these transport processes may occur simultaneously as well as sequentially, which is also reflected in the structure of the model (Fig. 1). However, the fraction of the segments oriented parallel $(1-a)$ and perpendicular (a) to the general direction of the heat flow is not known. The fraction (b) of the surface of the solid frame, which is soaked by moisture, is also not known.

Making use of the known relations for the effective thermal conductivity of layers connected in parallel and series, and also considering the known (or determined from experiment) parameters a and b, Krisher proposed the following sequence of computation of the thermal conductivity λ of a moist material:

$$\lambda = \frac{1}{\dfrac{1-a}{\lambda'_{\mathrm{m}}} + \dfrac{a}{\lambda''_{\mathrm{m}}}}, \tag{1}$$

where

$$\lambda'_{\mathrm{m}} = \lambda_S m_S + \lambda_W m_W + \lambda_{LD} m_{LD} + \lambda_L m_L,$$

$$\lambda''_{\mathrm{m}} = \frac{1}{\dfrac{m_S}{\lambda_S} + \dfrac{m_W}{\lambda_W} + \dfrac{m_{LD}}{\lambda_{LD}} + \dfrac{m_L}{\lambda_L}}, \tag{2}$$

$$m_S = 1 - m_P, \quad m_L = b(m_P - m_W), \quad m_{LD} = (1-b)(m_P - m_W). \tag{3}$$

It was assumed that parameter a could be determined from the system of equations [1]

$$\lambda_{\mathrm{dry}} = \frac{1}{\dfrac{1-a}{\lambda'_{\mathrm{dry}}} + \dfrac{a}{\lambda''_{\mathrm{dry}}}}, \quad \lambda_{\mathrm{sa}} = \frac{1}{\dfrac{1-a}{\lambda'_{\mathrm{sa}}} + \dfrac{a}{\lambda''_{\mathrm{sa}}}}, \tag{4}$$

where

$$\lambda'_{\mathrm{dry}} = \lambda_S(1-m_P) + \lambda_L m_P, \quad \lambda''_{\mathrm{dry}} = \frac{1}{\dfrac{1-m_P}{\lambda_S} + \dfrac{m_P}{\lambda_L}},$$

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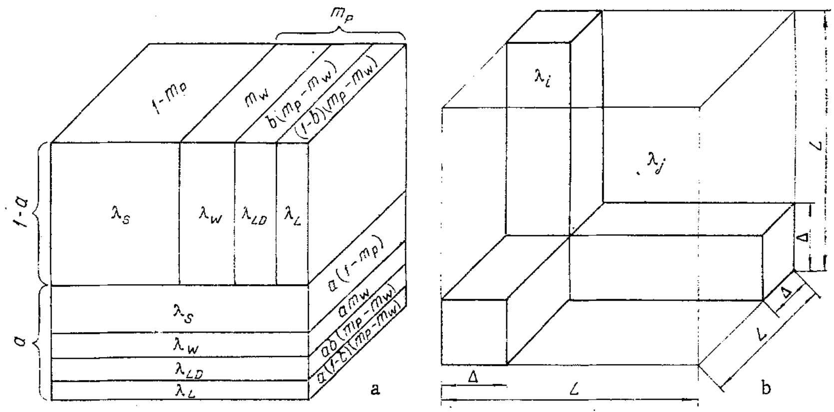

Fig. 1. Model of a moist material [1] (a) and an elementary cell of the interpenetrating system [2] (b).

$$\lambda'_{sa} = \lambda_S(1 - m_P) + \lambda_W m_P, \quad \lambda''_{sa} = \frac{1}{\dfrac{1 - m_P}{\lambda_S} + \dfrac{m_P}{\lambda_W}},$$

from known (measured) values of the thermal conductivity of dry porous (λ_{dry}) and completely saturated moist (λ_{sa}) material.

The thermal conductivity of the air — vapor mixture λ_{LD} was determined from the formula

$$\lambda_{LD} = \lambda_L + \lambda_D = \lambda_L + \frac{\delta}{R_D T} \cdot \frac{P}{P - P''_D} \cdot \frac{dP''_D}{dT} r. \tag{4a}$$

Thus, for the prognosis of the thermal conductivity of a moist porous material by the method of [1] the three empirical parameters b, λ_{dry}, λ_{sa} for each investigated material and a set of initial parameters λ_L, λ_W, λ_{LD}, m_P, m_W must be known.

A qualitative advantage of the model (Fig. 1) is the possibility of solving the inverse problem, i.e., determining a from known λ_{dry} and λ_{sa}.

In a later study, Misnar [2] made an attempt to construct a model of the structure of a moist porous solid material giving a better representation of its true structure than the parallel sheet model (Fig. 1a). Instead of the single model used in [1], Misnar proposed a set of two models and a new empirical parameter taking account of the contribution of each model to the effective thermal conductivity of the system as a whole; a critical review of Misnar's work is given in [3].

In the present article we propose to model the structure of a moist porous solid material by a structure with interpenetrating components [3]. This approach permits us to use a model giving a closer representation of the structure of real materials (Fig. 1b) and gets rid of the empirical parameter a; furthermore, it eliminates the need for measuring the thermal conductivity of completely moist λ_{sa} and dry λ_{dry} materials, which are computed by methods proposed in [3,4].

For this purpose a moist porous solid material is represented in the form of a multicomponent system; the number of the components, their thermal conductivity, and their concentration are the same as in [1]. In [3] a method of successive reduction to a two-component system is used for multicomponent systems, i.e., at the first stage a system consisting of the first and second components is considered, for example, a solid and dry air; the total volume of this system is reduced compared to the initial volume by the volume of the third and fourth components. Then the concentration of the second component in the separated volume is

$$m'_2 = \frac{V_2}{V - V_3 - V_4} = m'_L = \frac{V_L}{V - V_W - V_{LD}} = \frac{m_L}{1 - m_W - m_{LD}}. \tag{5}$$

The process of heat transport in the two-component model with interpenetrating components is investigated in an elementary cell of the system (Fig. 1b), since the effective thermal conductivity of the system as a whole is the same as that of its elementary cell [3].

The thermal conductivity of an elementary cell consisting of two components is computed from the formula

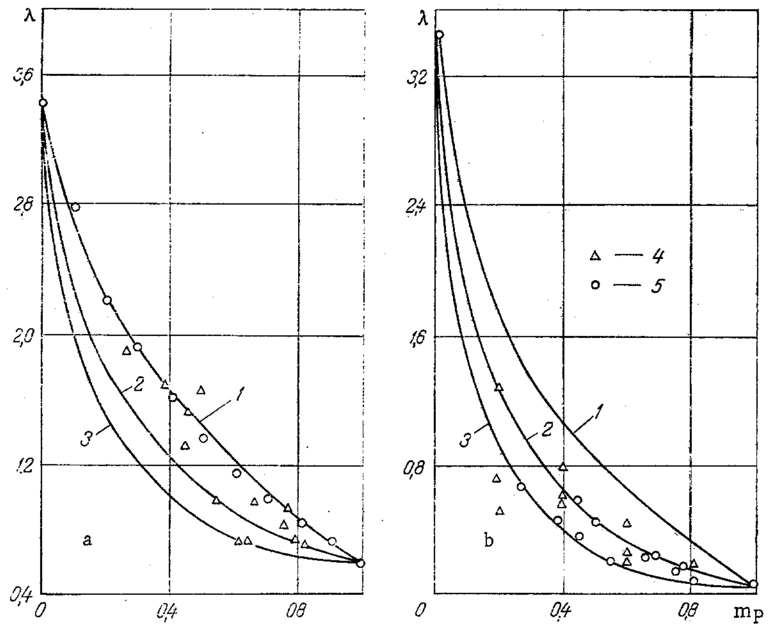

Fig. 2. Thermal conductivity of inorganic completely moist materials (a) and dry materials (b): 1) computed from formula (6); 2) computed from formula (8) for M = c; 3) computed from formula (8) for $M = c^2$; 5) experiment from [1]; 4) a — experiment from [5]; b — experiment from [3].

$$\lambda_{ij} = \lambda_i \left[c_j^2 + \nu_{ij}(1 - c_j)^2 + \frac{2\nu_{ij}c_j(1 - c_j)}{\nu_{ij}c_j + 1 - c_j} \right], \quad \nu_{ij} = \frac{\lambda_j}{\lambda_i}, \tag{6}$$

$$0 \leqslant m_j \leqslant 0.5 \quad c_j = 0.5 - \cos\left\{\frac{1}{3}\arccos(1 - 2m_2)\right\},$$

$$0.5 < m_j < 1.0 \quad c_j = 0.5 + \cos\left\{\frac{1}{3}\arccos(2m_2 - 1)\right\},$$

where at the first stage the index i refers to the solid component and the index j to the second, i.e., to dry air. Accordingly, we have

$$\lambda_i = \lambda_S, \ \lambda_j = \lambda_L, \ m_j = m_L', \ c_j = c(m_L'), \ \lambda_{ij} = \lambda_{SL}.$$

At the second stage we take the third component as the j component (water); the thermal conductivity of the i component is taken equal to λ_{SL} calculated from formula (6) at the first stage; the volume concentration of the third component (water) in the volume reduced by the volume of the fourth component is calculated from the formula

$$m_3' = \frac{V_3}{V - V_4} = m_W' = \frac{V_W}{V - V_{LD}} = \frac{\frac{V_W}{V}}{\frac{V}{V} - \frac{V_{LD}}{V}} = \frac{m_W}{1 - m_{LD}}. \tag{7}$$

The thermal conductivity of a three-component system consisting of the first (solid substance), second (dry air), and third (water) components λ_{SLW} is calculated from formula (6).

Finally at the third stage we take the air — vapor mixture contained in the pores with moist walls as the j component; the thermal conductivity of the j component is λ_{LD}, the volume concentration is m_{LD}, and the thermal conductivity of the i component is λ_{SLW}. The effective thermal conductivity of the entire system is also calculated from formula (6).

Thus, the seven parameters λ_S, λ_W, λ_L, λ_{LD}, m_P, m_W, b must be known for computing the thermal conductivity of moist systems in the first approximation, i.e., instead of the thermal conductivity of the dry and saturated moist materials it is sufficient to know the thermal conductivity of the base (thermal conductivity

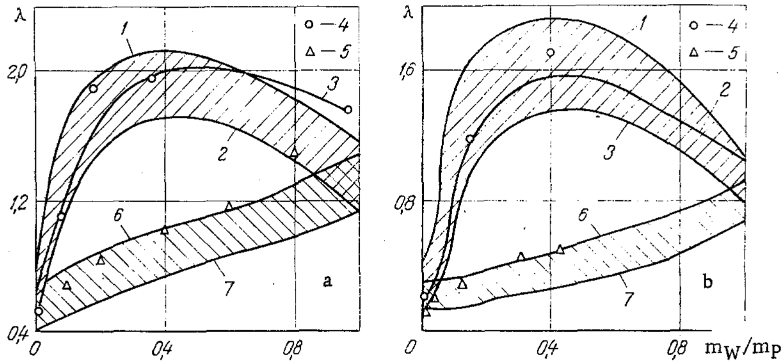

Fig. 3. Thermal conductivity: a) moist brick for m = 0.5 [1-4) t = 80°C]; b) moist Siporex for m = 0.78 [1-4) t = 82°C]: 1) computed from formula (6); 2) computed from formula (8) for M = c; 3) computed from formula (1); 4) experiment from [1]; 5-7) t = 20°C; 5) experiment from [1]; 6) from formula (6); 7) from formula (8) for M = c.

of the material with zero porosity). In [5] it is proposed that for many inorganic construction materials the thermal-conductivity coefficient may be taken approximately equal to 3.2 W/m · K. Hence, we do not need additional experimental data in the computation for moist construction materials of inorganic origin.

The thermal-conductivity coefficients of dry porous materials, materials whose pores are completely filled with water, and moist materials were calculated using the technique discussed above for a wide range of variation of the water content and temperature. The results are shown in Figs. 2 and 3.

The experimental data shown in Fig. 2a are taken from [1, 5] and agree well with the computational results for completely moist materials (curve 1) obtained from formula (6); this agreement is not observed for the dry materials. In the entire range of variation of porosity the experimental points lie appreciably below the values of the thermal conductivity computed from formula (6) (Fig. 2b).

Actually, inorganic porous construction materials usually have microfissures which sharply reduce the thermal conductivity of the material even though they have practically no effect on the porosity. The thermal conductivity of cracked materials can be computed by the method proposed in [3]:

$$\lambda = \lambda_i \left[c_j^2 M + \nu_{ij} (1 - c_j)^2 + \frac{2\nu_{ij} c_j (1 - c_j)}{\nu_{ij} c_j + 1 - c_j} \right], \tag{8}$$

where parameter M characterizes the increase in the thermal resistance of the solid component due to the fissures filled by the component with smaller thermal conductivity (gas, liquid). Parameter M depends on the porosity, the dimensions of the microfissures, and the thermal conductivity of the component filling the pores. The structure of parameter M is well substantiated in [3] for the model of a cracked material. However, the dimensions of the fissures are usually unknown; therefore, in the first approximation a rough estimate of this parameter can be obtained using the simplest relationship. Thus, the following formulas may be recommended for M for inorganic construction materials operating in an ordinary atmosphere: for weak fracturing M = c; for strong fracturing $M = c^2$.

Curves 1, 2, and 3 in Fig. 2 are computed taking M = f(c) for the cases M = 1, M = c, and $M = c^2$; curves 1 and 2 in Fig. 3 are for the cases M = 1 and M = c.

The nature of the dependence $\lambda = \lambda(m_W/m_P)$ is a reflection of the following physical process: on increasing the moisture content in the range $0 \leq m_W/m_P \leq 0.4$ the thermal conductivity increases mainly due to the increase of heat transport as a result of evaporation of moisture from wet walls; a subsequent increase of the moisture ($m_W/m_P > 0.4$) results in blocking of the pores by water drops and a consequent decrease of the transport of the vapor; for a completely moist material ($m_W/m_P = 1$), the heat transfer occurs only through the skeleton of the material and the pores filled by water.

The proposed methods of modeling the structure of moist porous solid construction materials make it possible to predict their thermal conductivity in a wide range of variation of the significant parameters without having to use partly empirical parameters and measure the thermal conductivity of dry and moist materials.

NOTATION

a, empirical parameter taking account of nonuniform sections oriented parallel and perpendicular to the general direction of heat flow in the model; b, empirical parameter taking account of the fact that only a part of the surface of the solid frame is wetted and contributes to the diffusive heat transfer due to evaporation and condensation of moisture; m_S, m_W, m_L, m_{LP}, volume concentration of the solid component, liquid, and pores with dry and wet walls, respectively; λ, effective thermal conductivity of moist materials; λ_S, λ_W, λ_L, λ_{LD}, thermal conductivity of the solid component, liquid, dry air, and air — vapor mixture (diffuse component); λ_{dry}, λ_{sa}, thermal conductivity of dry porous and completely moist material; δ, diffusion coefficient; P, total pressure of the mixture; P_D, saturated vapor pressure; R_D, universal gas constant; r, heat of evaporation.

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THERMAL-CONDUCTIVITY RANGE FOR A COMPOSITE HAVING KNOWN RANGES IN PARAMETERS FOR THE COMPONENTS

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The known ranges in thermal conductivity for the components may be used to determine the range in thermal conductivity for a composite; formulas have been derived for the distribution coefficients, which provide detailed values in each case.

There are presently many different methods of calculating thermal conductivities for composites in terms of the known conductivities of the components; these methods form the subject of several reviews [1-4].

In these methods it is assumed that the thermal conductivity and the degree of filling are known exactly, whereas in any measurement there is always some experimental error, and the final spread is governed by the error of measurement as well as by variations in the properties of the material itself. In either case, the measured value for the thermal conductivity is to be treated as a random quantity, one of the characteristics being the mathematical expectation (most likely value) and another being the standard deviation.

In this connection it is of interest to determine how the spread in the thermal conductivity for each of the components affects the spread in the same for the composite for various proportions of the components.

Further, a real composite also has a degree of filling in a finite volume that may also be considered as a random quantity, which deviates to some extent from the mean value. Therefore, the thermal conductivity of the composite should vary even within the volume of a specimen. We show below that in certain instances one can determine in simple fashion the spread in the thermal conductivity of the composite as a function of the spread in the degree of filling.

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